Research Article

Analytical Solution to Nonlinear Photovoltaic Diode Equation

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Abstract

Mathematical photovoltaic models are described by nonlinear equations which require iterative methods to obtain solutions. To curb computational complexity, this research presents an analytical method which estimates accurately, the model parameters of photovoltaic modules from the datasheet provided by manufacturers. This work adopts the general photovoltaic diode equation and introduces a new equation to solve for series resistance of the single diode photovoltaic system. Series resistance was determined at the maximum power point. The shunt resistance, photocurrent and saturation current were also determined. Results from the present analytical method are validated with those from other complex numerical methods in terms of computation. The results serve as a benchmark for experimental photovoltaic characterization.
Keywords: PV Model, Analytical Solution, Circuit Parameters.

Introduction

Photovoltaic (PV) energy industry has grown rapidly in recent years due to increasing need for green energy. Modern electricity generation technologies are built around renewable energy resources because of the numerous advantages associated with them such as non-emission of pollutants, renewability, inexhaustibility and the fact that they are generally noise-free [1]. Furthermore, escalating oil prices, global warming and environmental deterioration associated with existing electric power generation spurs the need for alternative energy resources [2]. Nonetheless, to maximize power extraction from PV systems, a good knowledge of PV module behavior based on manufacturers' datasheet information is essential. Unfortunately, PV module behaviors are described by nonlinear models which require tedious numerical procedures to obtain solution. For this reason, this paper presents a simple and straight forward, non-iterative analytical method for PV model parameters determination from datasheet information. This analytical method could enhance good parameterization of all elements used in PV models and as such serve as an invaluable tool in understanding the characteristics of a PV system as well as its application and performance in varied atmospheric conditions.

Different nonlinear models exist for PV systems. The single and multi-diode models which differ mainly on computational complexity. According to [3], the double diode model offers improved results as it takes into consideration the recombination loss at space depletion region of solar cells. Other researchers [4,5] proposed the three-diode model which analyzed the electrical characteristics of multi-crystalline solar cells and considers the effect of grain boundaries and leakage current through the surfaces. Generally, computational complexity increases with addition of extra diode beyond the single diode model. It is for this reason the single diode model is commonly used as a compromise between simplicity and accuracy. The rest of this paper is organized as follow: Section 2 describes the single- diode PV model while Section 3 presents the mathematical formulation of the analytical method. Results and discussion are presented in Section 4, and we conclude in Section 5.

Theoretical Background

Single- diode PV model

Figure 1 shows the single-diode model of the theoretical PV cell and equivalent circuit of a practical PV device including the series and parallel resistances. This consists of a diode, two resistors, namely, one series resistor (R_s) and one shunt resistor (R_p) and a constant current source [6]. PV parameters in this model include the photo-generated current (I_{pv}), diode saturation current (I_o),R_p,R_s and the diode ideality factor n. Several methods to extract the parameters of the single diode model exist in the literature. In [7], the parameters were obtained under atmospheric conditions. These authors proposed an approximation scheme which asserts that there is only one pair $\{R_s, R_p\}$ that establishes the equality of the model maximum power (P_{maxm}) and the experimental maximum power (P_{maxe}) at the maximum power point (MPP). Tayyan [8] proposed a set of five equations to extract the five parameters of the single diode model. Ma [9] proposed a polynomial approximation method to obtain the parameters of the single diode model. Other researchers [10], proposed an analytical method in which I_o,I_{pv}, R_p, and R_s were obtained and n chosen arbitrarily according to the general assumption in PV modeling that n exists in the range $1 \leq n \leq 1.5$ [7]. The current-voltage *(I-V)* characteristics of the single-diode PV model are given in [7] and shown in Figure 1.

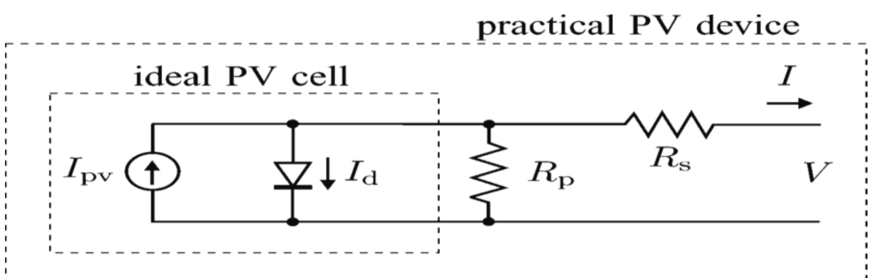

Figure1. Single-diode model of the theoretical PV cell and equivalent circuit of a practical PV device including the series and parallel resistances.

The current-voltage (*I-V*) characteristics of the single-diode PV model is given by equation (1):

$$I = I_{pv} - I_{o\left(e^{(V+IR_s/nV_T)}-1\right)} - \frac{V+IR_s}{R_p} \quad (1)$$

Performance estimation of equation (1) implies determination of five parameters, I_{pv}, I_o, R_s R_p and diode ideality factor n.
In equation (1), V_T is the thermal voltage of the diode, this constant depends on the ambient temperature T, Boltzmann constant k, N_s is the number of solar cells in series, and q is the electronic charge expressed with the equation (2):

$$V_T = \frac{N_s kT}{q} \quad (2)$$

Estimation Method

Mathematical formulation of the analytical method

The single- diode PV model described by equation (1) has five parameters to be determined: I_{pv}, I_o, R_s, R_p and n. The analytical method proposed in this work aims at determining these parameters using datasheet information. The datasheet provides three key points of the $I - V$ characteristic, namely, short-circuit current point; $I = I_{sc}$, $V = 0$, open-circuit voltage point; $I = O$, $V = V_{oc}$ and the maximum power point; $I = I_{mp}\, and\ V = V_{mp}$.
Following the simplifications in [10], their technique for determining I_{pv} were used to derive the expression for I_o in this work.

Analytical method for predicting $\mathbf{I_{pv}}$, $\mathbf{I_o}$ and $\mathbf{R_p}$

At the short-circuit current point $(I_{sc}, 0)$, equation (1) has the form:

$$I_{sc} = I_{pv} - I_o\left(e^{\frac{I_{sc}R_s}{nV_T}} - 1\right) - \frac{I_{sc}R_s}{R_p} \quad (3)$$

This simplifies to [10]:

$$I_{pv} = \frac{R_p + R_s}{R_p} I_{sc} \quad (4)$$

At the open- circuit voltage point $(0, V_{oc})$, substituting this in equation (1) gives:

$$0 = I_{pv} - I_o\left(e^{\frac{V_{oc}}{nV_T}} - 1\right) - \frac{V_{oc}}{R_p} \quad (5)$$

Using (4) in (5) and the fact that the term '-1' in (1) can be neglected for silicon devices [11], we obtain:

$$I_o = \frac{(R_p + R_s)I_{sc} - V_{oc}}{R_p e^{\frac{V_{oc}}{nV_T}}} \quad (6)$$

Also, at maximum power point (I_{mp}, V_{mp}), equation (1) takes the form

$$I_{mp} = I_{pv} - I_o\left(e^{\frac{(V_{mp} + I_{mp}R_s)}{nV_T}} - 1\right) - \frac{V_{mp} + I_{mp}R_s}{R_p} \quad (7)$$

Substituting equations (4) and (6) into (7), this simplifies to

$$I_{mp} = I_{sc} - \left(I_{sc} - \frac{V_{oc} - R_s I_{sc}}{R_p}\right) e^{\frac{(V_{mp} + I_{mp}R_s - V_{oc})}{nV_T}} - \frac{V_{mp} + I_{mp}R_s - I_{sc}}{R_p} \quad (8)$$

Now, from equation (8), solving for R_p results

$$R_p = \left(\frac{(V_{oc} - R_s I_{sc})e^{\frac{(V_{oc} + I_{mp}R_s - V_{oc})}{nV_T}} - (V_{mp} + R_s I_{mp}) + R_s I_{sc}}{\left(e^{\frac{(V_{mp} + I_{mp}R_s - V_{oc})}{nV_T}} - 1\right) I_{sc} + I_{mp}}\right) \quad (9)$$

From the above, equations (4), (6) and (9) give expressions for predicting I_{pv} , I_o and R_p respectively.

Series resistance function for extracting $\mathbf{R_s}$

In addition to STC specifications, PV datasheets also list values for current and voltage at maximum power point, namely, I_{mp} and V_{mp} respectively. This gives a way to estimating the numerical value for the experimental maximum power P_{maxe} . Similarly, the computed maximum power P_{maxm} in the present work is given by:

$$P_{maxm} = V_{mp}\left(I_{sc} - \left(I_{sc} - \frac{V_{oc}-R_S I_{sc}}{R_p}\right)e^{\frac{(V_{mp}+I_{mp}R_S-V_{oc})}{nV_T}} - \frac{V_{mp}+I_{mp}R_S-I_{sc}}{R_p}\right) \quad (10)$$

Now, equation (10) in terms of resistances satisfies the relation given in equation (11):

$$P_{maxm} = f\left(R_s, R_p\right) \quad (11)$$

To obtain the optimal maximum power for the model, a function of one variable is required according to the maximum power transfer theorem which asserts that the source resistance that maximizes power transfer is always zero regardless of the load resistance [12].

Now, in PV modeling, it is generally assumed that $R_s \to 0$and $R_p \to \infty$[7] and gives $f(R_s)$ for equation (11) when this condition is applied to equation (10). Therefore, the optimal model power is given by equation (12):

$$P'_{maxm} = f(R_s) = V_{mp}\left[I_{sc} - I_{sc}e^{\frac{(V_{mp}+I_{mp}R_S-V_{oc})}{nV_T}}\right] \quad (12)$$

Equation (12) is the series resistance function proposed in this work. To obtain R_s , equation (13) must be satisfied for any value of R_s chosen arbitrarily from R_s= 0, 0.1, 0.2, 0.3, …

$$P_{maxe} = p'_{maxm} \quad (13)$$

In this scheme, the value of $R_s$which satisfies equation (13) is the numerical value for R_s in the present method.

Resolution process

Equations (4), (6), (9) and (13) are a set of decoupled equations representing I_{pv}, I_o , R_P and R_s respectively.

The resolution process for calculating these parameters from the three key operating points at a given temperature is as follows:

For the purpose of this research, the parameter n is estimated as the mean value within the range $1 \leq n \leq 1.5$. This range is established in [7].

i. Determine R_s from equation (13)
ii. Determine R_p from equation (9)
iii. Determine I_o from equation (6)
iv. Determine I_{pv} from equation (4)

Table1. Parameters of Mitsubishi and KC200GT solar panels at STC

Parameters	Mitsubishi	KC200GT
I_{mp}	6.93A	7.61A
V_{mp}	24.6V	26.3V
P_{maxe}	170.478W	200.143W
I_{sc}	7.38A	8.21A
V_{oc}	30.6V	32.9V
N_s	50	54
T	25℃	25℃

Results and Discussion

Analytical method to determine R_s, R_p, I_{pv} and I_o of the single diode PV model is presented. Mitsubishi [13] and KC200GT [14] solar panels with manufacturers datasheet information in Table 1 was examined and results are shown in Table 2. The values for V_{mp} and I_{mp} of the present method are determined by adopting the validation method in [7] for which $P_{maxm} = V_{mp}I_{mp}$ at MPP (V_{mp}, I_{mp}). These values agree perfectly with those provided by the manufacturers and as such make Table 2 a sort of reference table for PV characterization.

In Tables 3-4 a comparison between the calculated values of the analytical method and published values from reference methods is shown. As can be inferred from Table 3, it is evident that the model parameters obtained from the analytical method agree with the parameters obtained by [8] and show a little deviation with values obtained by [10] and [7]. This notwithstanding, model parameters values from both the analytical and reference method depict the same other of magnitude and as such justify the accuracy of the present method. In Table 4, both the analytical method and [15] give model parameters in the same order of magnitude. A deviation is observed for $\boldsymbol{I_o}$.

Table2. Parameters of the analytical method of Mitsubishi and KC200GT

Parameters	Mitsubishi	KC200GT
I_{mp} {A}	6.93	7.61
V_{mp} {V}	24.58	26.44
P_{maxm} {W}	170.340	201.220
I_o{A}	$3.820 \cdot 10^{-8}$	$4.620 \cdot 10^{-8}$
I_{pv}{A}	7.382	8.215
n	1.25	1.25
R_p{Ω}	672.627	443.977
R_s{Ω}	0.200	0.250

Table 3. Comparison between the calculated values of analytical method and the published values of the parameters from references for KC200GT solar panel

Parameters	Analytical method	[7]	[10]	[8]
n	1.25	1.30	1.30	1.25
I_o{A}	$4.620 \cdot 10^{-8}$	$9.825 \cdot 10^{-8}$	$9.699 \cdot 10^{-8}$	$4.812 \cdot 10^{-8}$
I_{pv}{A}	8.215	8.214	8.213	8.215
R_s { Ω }	0.250	0.221	0.231	0.247
R_p{ Ω }	443.977	415.405	594.851	414.890

Table 4. Comparison between the calculated values of the analytical method and the published values of the parameters from reference methods for Mitsubishi solar panel

Parameters	Analytical method	[10]	[15]
n	1.25	1.10	1.30
I_o{A}	$3.820 \cdot 10^{-8}$	$2.880 \cdot 10^{-9}$	$8.580 \cdot 10^{-8}$
I_{pv}{A}	7.382	7.382	7.378
R_s{ Ω }	0.200	0.288	0.211
R_p{ Ω }	672.627	1361	10570

This paper has shown a simple and straight forward, non-iterative analytical method which estimates accurately the nonlinear PV model parameters from the datasheet provided by PV manufacturers. In this work, a new equation:

$P'_{maxm} = f(R_s) = V_{mp}\left[I_{sc} - I_{sc}\, e^{\frac{(V_{mp} + I_{mp} R_s - V_{oc})}{nV_T}}\right]$ at MPP was introduced.

This equation allows the easy determination of the model parameters from a set of decoupled equations and as such curbs the computational complexity associated with existing methods in the literature. The simplicity of the present analytical method makes it a procedure for a number of applications most especially for end users with limited computational resources.

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