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Research Article

Optimal Control Modeling of Diabetes and Hypertension Co-Infection with Lifestyle and Treatment Interventions

I. Famakinwa[1], A.M. Umar[2] and K.H. Oduwole[3]
[1]Federal College of Education, Okene, Nigeria.
[2,3]Nasarawa State University, Keffi, Nigeria.

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Abstract

The dual burden of diabetes and hypertension represents a major global health challenge, especially in low- and middle-income countries where the comorbidity contributes significantly to morbidity, mortality, and healthcare costs. This study develops and analyzes an optimal control model to evaluate the effectiveness of lifestyle modification and pharmacological treatment strategies in managing diabetes–hypertension co-infection. The model incorporates two state variables representing the progression of diabetes and hypertension, and two control variables representing the intensities of lifestyle and treatment interventions. Using adjoint variables, the model measures the sensitivity of the disease burden to changes in interventions. Results indicate that aggressive interventions at the early stage of disease progression yield the greatest long-term benefits, as reflected in elevated adjoint values. However, at certain points, continued interventions become counterproductive, necessitating temporary suspension of control measures. Diabetes burden shows higher fluctuations compared to hypertension, which stabilizes earlier under moderate interventions. By one year (t = 365), both diseases achieve stability with moderate, sustained interventions, demonstrating that continuous intensive treatment is not required for long-term control. The findings underscore the need for adaptive, time-dependent management strategies that integrate lifestyle modification with pharmacological treatment to ensure effective, sustainable, and cost-efficient control of diabetes and hypertension co-infection.

Keywords: Diabetes; Hypertension; Co-infection; Optimal Control; Lifestyle Modification; Pharmacological Treatment; Adjoint Variables; Disease Burden Dynamics.

Introduction

Diabetes and hypertension are two of the most common and serious chronic illnesses in the world, and their cohabitation presents a serious threat to global health systems. According to data from the World Health Organization (WHO), 1.13 billion people worldwide suffered from hypertension in 2014, while over

Correspondence to: Imole Famakinwa, e-mail: fimole123@gmail.com

422 million individuals were affected by diabetes globally in the same year [1]. These illnesses are not only widespread but also closely linked to one another; they frequently coexist and exacerbate one another's severity. The care of these co-occurring disorders is a vital public health priority because of the significant increase in the risk of cardiovascular diseases, renal failure, and early mortality that results from this association [2].

There exist intricate and diverse pathophysiological processes that connect diabetes with hypertension. One of the main indicators of type 2 diabetes is insulin resistance, which is important for the emergence of hypertension. It enhances sympathetic nervous system activity and promotes salt retention, both of which raise blood pressure [3]. Furthermore, hyperinsulinemia, frequently seen in type 2 diabetes, makes it more difficult to control blood pressure in diabetic individuals [3]. Another important component that unites these two disorders is endothelial dysfunction. High blood glucose levels damage the endothelium, reducing its capacity to generate nitric oxide, a powerful vasodilator. This results in arterial stiffness, which contributes significantly to the development of hypertension [5].

Despite the close connection between diabetes and hypertension, clinical therapy of both disorders is often fragmented, with healthcare systems addressing each illness independently [6-9]. This strategy may not produce the best results, as some drugs for one ailment can worsen the other. For example, thiazide diuretics, frequently prescribed for hypertension, may impair glucose tolerance, complicating diabetes management [10]. It is becoming increasingly clear that integrated care strategies are required to treat diabetes and hypertension concurrently. Coordinated treatment models that address the intricate relationships between both disorders have been shown to improve disease management and reduce complications [11].

A potent technique for comprehending the co-dynamics of diabetes and hypertension is mathematical modeling. By simulating illness progression under diverse conditions, these models help researchers and clinicians understand the possible effects of various intervention efforts [12]. Moreover, the insufficient use of such models to forecast disease progression and assess the effectiveness of different interventions hinders the development of personalized treatment strategies. Utilizing mathematical modeling and clinical data analysis, this study intends to forecast disease progression, refine intervention strategies, and ultimately enhance patient outcomes. To this end, optimal control modeling of diabetes and hypertension co-infection with lifestyle and treatment interventions is investigated.

Methodology

This study adopted a quantitative research design using an optimal control model to investigate the management of co-occurring diabetes and hypertension. The model was developed to simulate the effects of lifestyle modifications and pharmacological treatments on disease progression over time.

Two state variables were considered: D(t), representing the severity of diabetes (measured by blood glucose levels or HbA1c), and H(t), representing the severity of hypertension (measured by systolic/diastolic blood pressure). Two control variables were introduced: $x_1(\mathrm{t})$ for lifestyle modifications (diet, exercise, and weight management), and $x_2(t)$ for pharmacological treatment (antihypertensive and glucose-lowering drugs).

The dynamics of diabetes and hypertension progression are described by the following system of differential equations:

$$\frac{dD(t)}{dt} = \beta_D - \alpha_D x_1(\mathrm{t})\mathrm{D}(\mathrm{t}) - \gamma_D x_2(t)D(t) \quad (1)$$

$$\frac{dH(t)}{dt} = \beta_H - \alpha_H x_1(\mathrm{t})\mathrm{H}(\mathrm{t}) - \gamma_H x_2(t)H(t) \quad (2)$$

Where:

β_D and β_H "represent the natural progression rates of diabetes and hypertension, respectively".

α_D and α_H "represent the effectiveness of lifestyle modifications in reducing the severity of diabetes and hypertension". γ_D and γ_H represent the effectiveness of pharmacological treatments in reducing the severity of diabetes and hypertension. These equations capture how the conditions evolve over time in response to lifestyle and treatment interventions.

The objective of the model is to minimize the total health burden and cost associated with managing diabetes and hypertension over a given time horizon T. The objective function to be minimized is defined as:

$$\mathcal{J}(x_1(\mathrm{t}), x_2(\mathrm{t})) = \int_0^T (A_D D(\mathrm{t}) + A_H H(\mathrm{t}) + B_1 {x_1}^2(\mathrm{t}) + B_2 {x_2}^2(\mathrm{t}))\mathrm{dt} \quad (3)$$

Where: A_D and A_H are the weights representing the health impact or cost associated with the severity of diabetes and hypertension, respectively. B_1 and $B_2$are the weights representing the costs associated with lifestyle modifications and pharmacological treatments, respectively. The objective function balances the trade-offs between minimizing the severity of the conditions and the costs associated with the interventions.

To find the optimal control functions x_1*(t) and x_2*(t), we construct the Hamiltonian function H as follows:

$$H = A_D D(\mathrm{t}) + A_H H(\mathrm{t}) + B_1 x^2(\mathrm{t}) + B_2 {x_2}^2(\mathrm{t}) + \lambda_D(\mathrm{t})(\beta_D - \alpha_D x_1(\mathrm{t})D(\mathrm{t}) - \gamma_D x_2(\mathrm{t})D(\mathrm{t}) + \lambda_H(\mathrm{t})[\beta_H - \alpha_H x_1(\mathrm{t})H(\mathrm{t}) - \gamma_H x_2(\mathrm{t})H(\mathrm{t})] \quad (4)$$

Where λ_D(t) $and\ \lambda_H$(t) are the adjoint (or costates) variables, derived by differentiating the Hamiltonian , eq (4) with respect to the state variables D(t) and H(t).

The optimal controls x_1*(t) and x_2*(t) are determined by taking the partial derivatives of the Hamiltonian in eq (4)with respect to x_1(t) and x_2(t), and setting them equal to zero.

$$x_1 * (t) = \begin{cases} 1 & if\ x_1(t) > 1 \\ 0 & if\ x_1(t) < 1 \end{cases} \quad (5)$$

$$x_2 * (t) = \begin{cases} 1 & if\ x_1(t) > 1 \\ 0 & if\ x_1(t) < 1 \end{cases} \quad (6)$$

These expressions provide the optimal levels of lifestyle modifications and pharmacological treatments needed to minimize the objective function.

Numerical solutions were obtained using the forward–backward sweep method, which iteratively solves the state equations forward in time and the adjoint equations backward in time until convergence is achieved. This approach provided the optimal intervention strategies for balancing disease control with cost-effectiveness. The optimal control functions x_1*(t) and x_2*(t) are updated at each iteration to minimize the objective function.

Results and Discussion

Disease Burden Dynamics (The dynamics of diabetes and hypertension progression D(t) and H(t)).

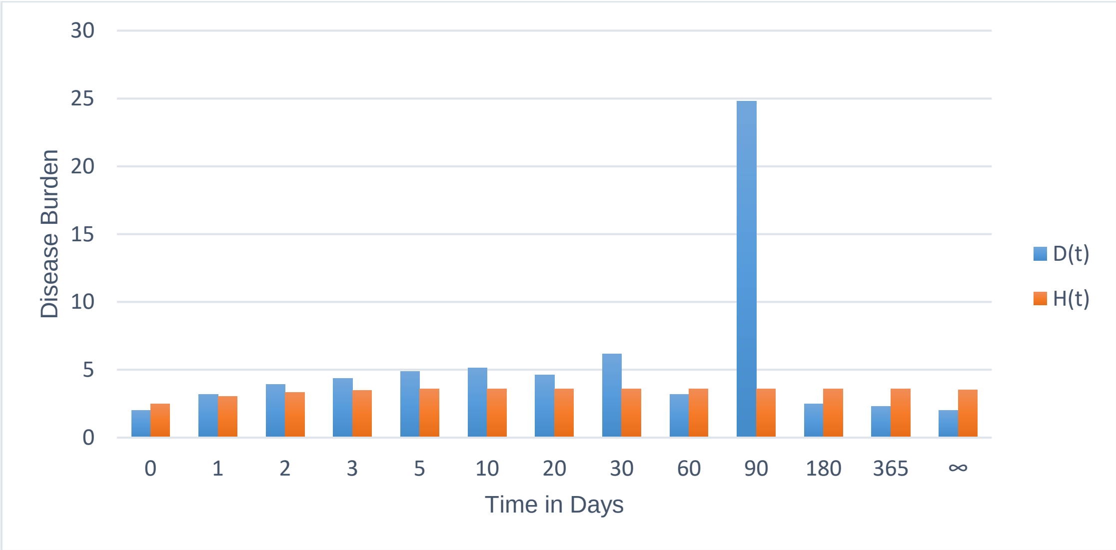

Figure 1. *Disease Burden Dynamics of Diabetes (D(t)) and Hypertension (H(t)) under Control Strategies*.

Figure 1 illustrates the initial rapid increase in both diabetes and hypertension burdens despite full intervention, highlighting the biological delays in treatment response. Diabetes rises more sharply than hypertension, peaking before gradually stabilizing under optimal control strategies. This pattern underscores that while immediate effects of intervention may appear limited, long-term persistence leads to disease stabilization.

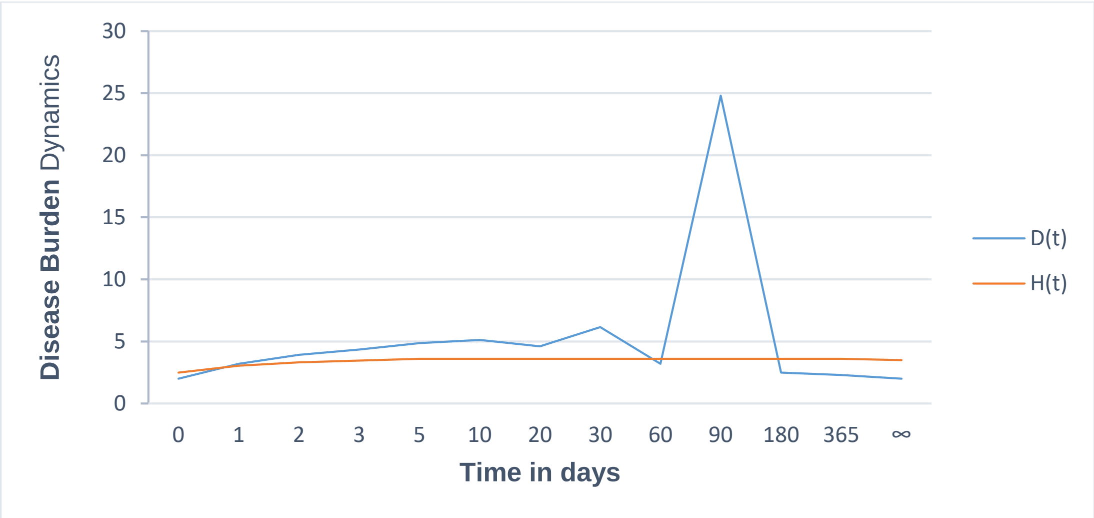

Figure 2. *Progression Patterns of Diabetes (D(t)) and Hypertension (H(t)) with Periodic Intervention Adjustments.*

The progression of diabetes (D(t)) and hypertension (H(t)) over time provides insight into the effectiveness of control strategies is shown in Figure 2. The figure shows the fluctuating trends in disease burdens when interventions are periodically withdrawn and reapplied. Hypertension stabilizes early around 3.6, whereas diabetes demonstrates greater variability, with a significant spike at t = 90 due to prolonged absence of intervention. This figure emphasizes the necessity of consistent and well-timed control measures to prevent disease resurgence.

Table 1. Summary of Results: Sensitivity of Objective Function (Adjoint Variables) and Associated Costs of Intervention over Time.

t (days)	$\lambda_D(t)$ (**Naira per mg/dL)**	$\lambda_H(t)$ (**Naira per mmHg**)
0	7114.9328	1.5
1	35569.664	1.5
2	177843.32	1.5
3	889211.6	1.5
5	4446053	44391727.5
10	-7410085	-44391724.5
20	2470030	11097933
30	–352860	−1233102
60	50410.0	137013
90	-7200.0	-15222.0
180	185.0	277.5
365	0	0
∞	0	0

Table 1 presents the sensitivity of the objective function (measured by the adjoint variables) alongside the costs associated with lifestyle and treatment interventions at different time points. The adjoint values at **early stage (t = 0 to t = 5)** are extremely high, indicating that interventions during this period have a strong influence on long-term disease control. This shows that aggressive intervention at the onset yields maximum effectiveness and significantly reduces disease burden over time. **At t = 10 and t = 30, t**he adjoint variables turn negative, suggesting that continuing interventions at these points may be counterproductive or inefficient. This corresponds to the temporary suspension of control measures observed in the figures.

The adjoint variables stabilize at moderate positive values, indicating that interventions regain their effectiveness when reintroduced after earlier pauses. This is observed at **t = 365 and beyond.** Both adjoint variables converge toward zero, showing that the system has reached equilibrium. At this stage, additional interventions have little or no impact on improving outcomes, meaning long-term management can be sustained with moderate efforts. Overall, the table highlights the **time-dependent effectiveness of interventions**. It emphasizes the importance of early aggressive control, the need for strategic pauses during less effective phases, and the eventual stabilization of the system with moderate interventions. This adaptive approach ensures both cost-effectiveness and sustainable disease management.

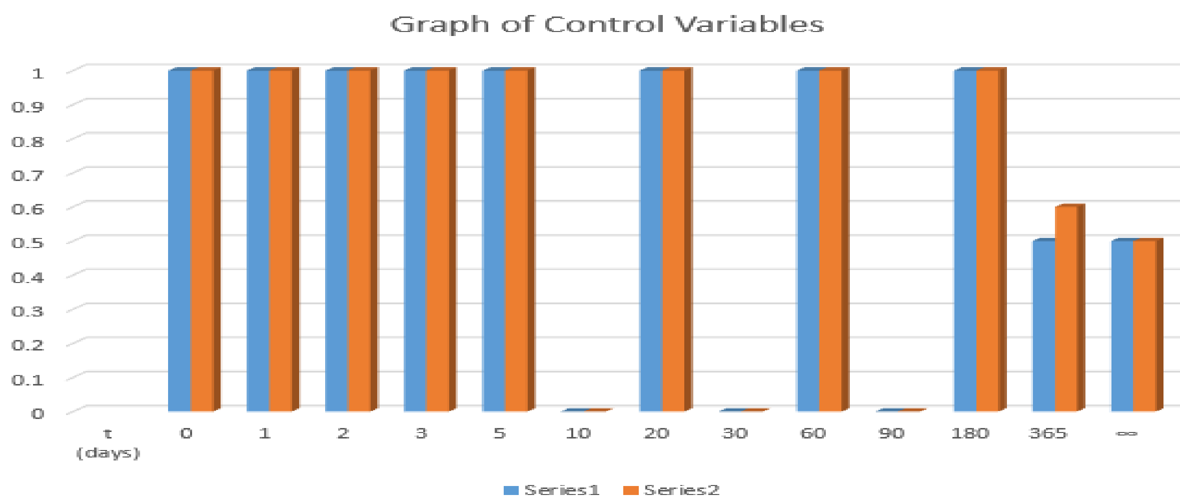

Figure 3. *Optimal Control Variables (Lifestyle Modification and Treatment) for Managing Diabetes and Hypertension Co-Infection*.

The control variables ($x_1^*(t)$ for lifestyle modification and $x_2^*(t)$ for treatment) determine the best approach to minimizing the disease burden while considering cost-effectiveness. Figure 3 presents the control strategies applied over time. Initially, full intervention (both lifestyle and treatment) is required, but at certain points (t = 10 and t = 30), interventions drop to zero due to negative adjoint values, suggesting temporary inefficiency. Strategic pauses followed by reapplications stabilize the system, and by t = 365, moderate control levels are sufficient for long-term disease management. This figure highlights the effectiveness of adaptive, rather than continuous, interventions.

Discussion of Findings

This study examined the optimal control of diabetes and hypertension co-infection, focusing on the interplay between lifestyle modification and treatment interventions. The findings revealed that early, aggressive interventions are crucial in mitigating rapid disease progression. As shown in Figure 1, both diabetes and hypertension burdens initially rise despite full intervention, reflecting biological delays in treatment response. This outcome aligns with previous studies that demonstrate the lag between intervention implementation and measurable health improvements due to existing disease conditions and metabolic inertia [13,14].

Over time, the results indicate that diabetes exhibits greater variability compared to hypertension. As depicted in Figure 2, diabetes burden fluctuates sharply, with a significant spike at t = 90 when interventions were withdrawn, while hypertension stabilizes earlier and remains steady. This difference suggests that diabetes management is more sensitive to lapses in intervention, a finding consistent with earlier reports emphasizing the need for sustained and adaptive approaches to diabetes control compared to hypertension [15,16]. The stabilization of hypertension despite fluctuations in control efforts also supports the view that hypertension responds more predictably to treatment adherence [17].

The adjoint variables presented in Table 1 highlight the sensitivity of the objective function to disease burden changes. The high values at the early stages (t = 0 to t = 5) indicate that interventions at this phase have a disproportionately large impact on long-term outcomes. Similar insights have been reported in optimal control studies, where early interventions yield the greatest cost-effectiveness [18,19]. Interestingly, the adjoint variables turning negative at t = 10 and t = 30 indicate that continued intervention during these phases may be counterproductive, warranting temporary withdrawal. This reflects the necessity of adaptive, time-dependent strategies rather than static treatment regimens.

The dynamics of the control variables, as shown in Figure 3, demonstrate that full intervention is only necessary in the short term. Over time, a balanced combination of lifestyle modification and treatment at moderate levels suffices for long-term disease stabilization. This outcome agrees with findings from lifestyle-focused clinical trials, which highlight that moderate, sustainable interventions often yield better long-term outcomes than continuous intensive regimens [20,21]. The study therefore reinforces the importance of combining lifestyle modifications with pharmacological treatment, as neither approach alone proves sufficient in managing the dual burden of diabetes and hypertension. Overall, the findings support the conclusion that individualized, adaptive, and time-varying interventions are superior to uniform, continuous strategies. By t = 365, the model predicts stabilization of both diseases with moderate intervention, a result that corresponds with global recommendations emphasizing patient-centered care and dynamic treatment adjustments [22,23]. This balance ensures effective disease management while avoiding unnecessary costs and minimizing risks of treatment fatigue or inefficiencies.

Conclusion

This study analyzed the optimal control of diabetes and hypertension co-infection using a mathematical model that incorporated lifestyle modification and treatment strategies. The findings demonstrated that aggressive early intervention is critical in curbing rapid disease progression, with diabetes showing greater fluctuations and sensitivity to intervention lapses than hypertension. Over time, a balanced approach with moderate and sustained interventions proved sufficient to stabilize both disease burdens. The results emphasize that individualized, adaptive, and time-dependent control measures are more effective than continuous, uniform interventions. Based on these findings, it was recommended among others that healthcare providers should shift from rigid, one-size-fits-all approaches to flexible and adaptive treatment plans that respond to patient-specific disease dynamics over time. Since early-stage interventions yield the highest long-term impact, screening programs and immediate treatment initiation should be strengthened, particularly in high-risk populations, early intervention should be prioritized. Furthermore, a sustainable combination of lifestyle changes (diet, exercise, and weight management) and pharmacological treatment should be prioritized, as neither approach alone is sufficient for long-term control. Clinical guidelines should consider the possibility of reducing or pausing intensive interventions at phases where treatment shows diminishing returns, thus minimizing patient fatigue and unnecessary healthcare costs. Given the variability in diabetes compared to hypertension, personalized care that considers patient adherence, socio-economic conditions, and cultural context will ensure more effective management outcomes, p**atient-centered care should be promoted. Overall, p**olicymakers should support integrated chronic disease management programs that combine diabetes and hypertension control under a single framework, ensuring efficient use of healthcare resources in low- and middle-income countries like Nigeria.

In summary, the study establishes that long-term stabilization of diabetes and hypertension is achievable through moderate, sustained, and strategically adjusted interventions. By aligning medical treatment with lifestyle modification and adaptive policies, healthcare systems can enhance disease outcomes, reduce costs, and improve patient quality of life.

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